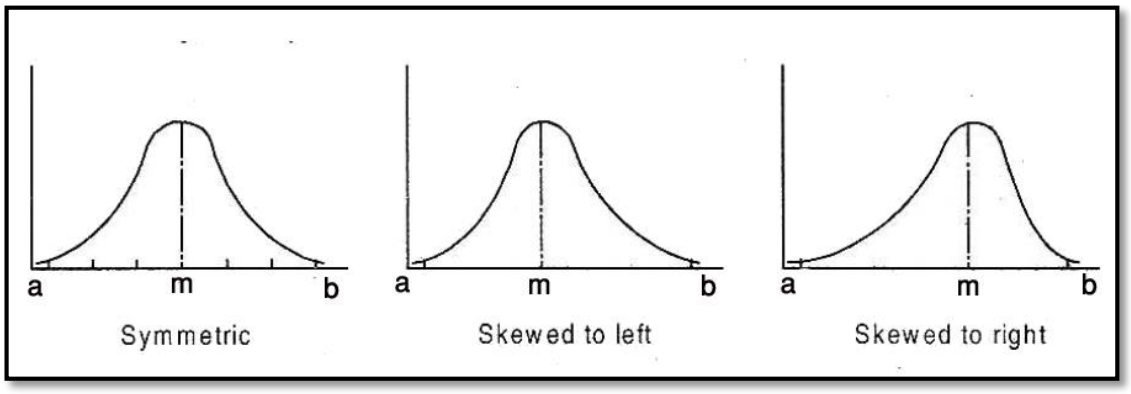
Project Evaluation and Review Technique (PERT) Method

PERT differs from CPM in that it assumes **probabilistic duration times** based on three estimates while the time estimates for CPM are deterministic.

1- **Optimistic time, a,** and estimate of the activity's duration under the most favorable conditions which is the shortest possible time required for the completion of an activity.
2- **Pessimistic time, b,** and estimate of the activity's duration under the worse conditions which is the maximum possible time the activity will take if everything goes bad.
3- **Most likely time, m,** most likely value for the activity duration and estimate under normal conditions. Value of m falls in the range (a, b).

The best way of predicting the three time estimates is by intelligent guessing. The experienced person who may be an engineer, foreman or worker having sufficient technical competence is asked to guess the various time estimates. The probabilistic nature of the activity times is described by the beta distribution. The three estimates (a, b, m) will be determined the mean and variance of beta distribution.



Based on the estimates, the mean (expected time) **average duration time**, $\overline{D}$, and **variance**, v, are approximated as

$$\overline{D} = \frac{a + b + 4m}{6}$$

$$v = \sigma^2 = \left(\frac{b-a}{6}\right)^2$$

Notes for solve by PERT method

1- Prepare a list of all activities (Time, human resources, money, and material) involved in the project.

2- Draw a PERT diagram as per the rules discussed earlier for drawing a network diagram.

3- Number all events in the ascending order from left to right.

4- Estimated average activity time is $\overline{D}_{ij}$ and the variance v_{ij}.

5- Find the Critical Path CP which is the total duration of the activities.

6- Find the variance v of each activities time of the CP.

7- Find the standard deviation of critical path $\{v\ (CP)\}$ is:

$$\sigma = \sqrt{v(CP)} = \sqrt{\sum_{i=1}^{n} v_i}$$

where v_i is variance of critical activities.

Let CP be the random variable denoting that the total duration of the activities on a critical path. Applying the assumption that CP is normally distributed, we can answer questions such as the following: What is the probability that the project will be completed within certain time

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Estimation on project completion time using standard normal variance value.

Calculate the probability of completing the project or before a specified completion date by using the standard normal equation shown below:

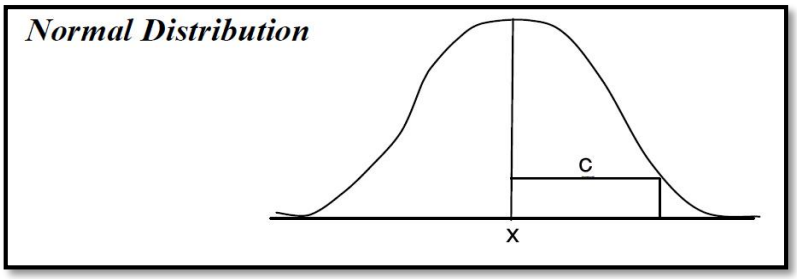
$$C = \left(\frac{K-X}{\sigma}\right)$$

Where

K = Proposed project completion time.

X = Expected time for critical path.

σ = the standard deviation of critical path.



The probability that the project will be completed in CP duration time is

P(project finishes within K time) = **P(CP ≤ X) = P(Z ≤ C)**
Where Z is a random variable with standard normal distribution

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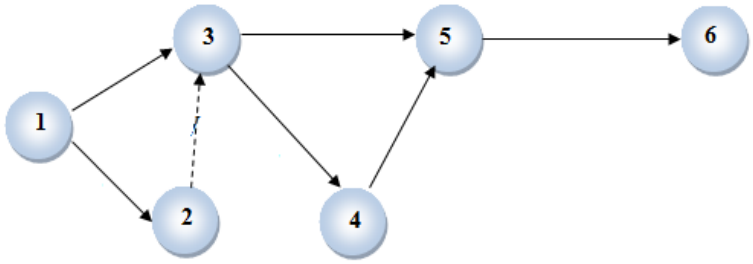
Example1: Consider the following table with **a, b, m** for activities of the network down. Find the probabilities that the different nodes of the project are realized without delay? What is the probability that the project will be completed within 35 days?



activities	a	b	m
1-2	**5**	**13**	**9**
1-3	**2**	**10**	**6**
3-5	**3**	**13**	**8**
3-4	**1**	**13**	**7**
4-5	**8**	**12**	**10**
5-6	**9**	**15**	**12**

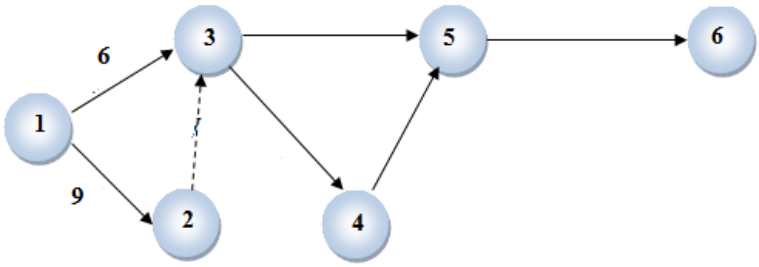
Solution:

1- Find average activities time and variance for all the activities.

activities	$\bar{D}_{ij}$	v_{ij}
1-2	**9**	**1.78**
1-3	**6**	**1.78**
3-5	**8**	**2.78**
3-4	**7**	**4**
4-5	**10**	**0.44**
5-6	**12**	**1**

Note that dummy activity with (a, b, m) = (0, 0, 0) has zero mean and variance. So $\bar{D}_{23} = v_{23} = 0$.

A network diagram with the average activities time is:



2- Determine the critical path for the project.

Critical Path: 1→2→3→4→5→6 $\Rightarrow$ (has longest path)

3- Determine the expected completion time for the project.

$\bar{D}$ For CP = 9 + 0 + 7 + 10 + 12 = 38

The expected completion time for the project is =38 days

4- Find variance for each activity for the CP for the project.

$V = V(c.p) = \sum V_{ij}$, V_{ij} is variance of critical activities

Variance for CP = 1.78 + 0 + 4 + 0.44 + 1 = 7.22

5- Find Standard deviation of critical path.

The standard deviation for CP is

$$\sigma = \sqrt{7.22} = 2.69.$$

6- **The probability that the project will be completed within 35 is just P(CP ≤ 35) days.**

K = 35

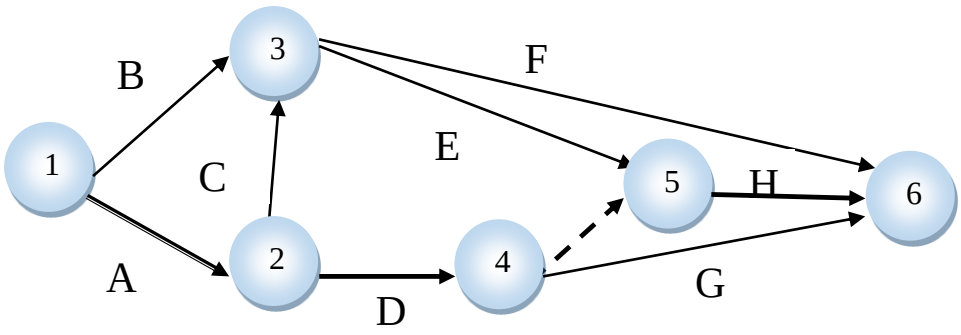
X = 38

σ = Standard deviation of critical path = 2.69

$$C = \left(\frac{K - X}{\sigma}\right) = \left(\frac{35 - 38}{2.69}\right) = -1.12$$

P (CP ≤ 35) = P (Z ≤ -1.12) = 0.1335 (from normal distribution table)
This means that there is a 13 .35% chance that the project will be completed within 35 days.

Example2: Consider the following project with the estimates **a, b, m** for activities. Find the probabilities that the different nodes of the project are realized without delay. What is the probability that the project will be completed within 20 days? 30 days?



activities	a	b	m
1-2	**3**	**7**	**5**
1-3	**4**	**8**	**6**
2-3	**1**	**5**	**3**
2-4	**5**	**11**	**8**
3-5	**1**	**3**	**2**

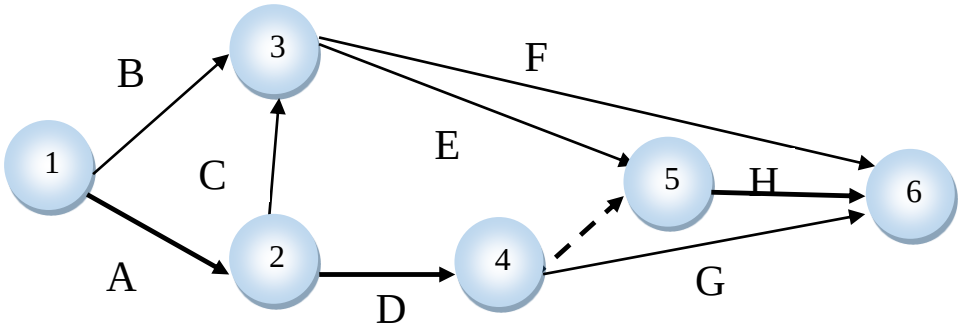
3-6	**9**	**13**	**11**
4-6	**1**	**1**	**1**
5-6	**10**	**14**	**12**

Solution:

1- Find average activities time and variance for all the activities.

activities	$\overline{D}_{ij}$	v_{ij}
1-2	**5**	**0.444**
1-3	**6**	**0.444**
2-3	**3**	**0.444**
2-4	**8**	**1.000**
3-5	**2**	**0.111**
3-6	**11**	**0.444**
4-6	**1**	**0.000**
5-6	**12**	**0.444**

Note that $\overline{D}_{45} = v_{45} = 0$. A network diagram with the average activities time is:



1- Determine the critical path for the project using the average activities time.

Critical Path: 1→2→4→5→6 ⇒ (has longest path)

2- Determine the expected completion time for the project.

$\overline{D}$ For CP = 5 + 8 + 0 + 12 = 25
The expected completion time for the project is = 25 days

3- Find variance for each activity for the CP for the project.

$V = V(c.p) = \sum V_{ij}$, V_{ij} is variance of critical activities

Variance for CP = 0.444 + 1 + 0 + 0.444 = 1.888

4- Find Standard deviation of critical path.

The standard deviation for CP is

$$\sigma = \sqrt{1.888} = 1.37$$

The probability that the project will be completed within 30 is just P(CP ≤ 30) days.

K = 20
X = 25
σ = Standard deviation of critical path = 1.37

$$C = \left(\frac{K - X}{\sigma}\right) = \left(\frac{20 - 25}{1.37}\right) = -3.6493$$

P (CP ≤ 20) = P (Z ≤ -3.649) = 0.0002 (from normal distribution table). This means that there is a 0.02 % chance that the project will be completed within 20 days.

The probability that the project will be completed within 30 is just P(CP ≤ 30) days.

$$C = \left(\frac{K - X}{\sigma}\right) = \left(\frac{30 - 25}{1.37}\right) = 3.6493$$

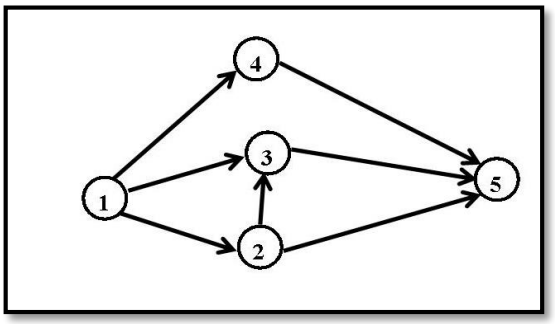
P (CP ≤ 30) = P (Z ≤ 3.649) = 0.9998 (from normal distribution table)

This means that there is a 99.98% chance that the project will be completed within 30 days.

1- **H.W: C**onstruct a network diagram for a project consisting the following estimates of activity times (weeks). What is the probability that the project will be completed within 18 weeks?, 11 weeks?

activities	pessimistic time b	optimistic time a	most probable time (m)
1-2	**6**	**2**	**4**
1-3	**7**	**6**	**5**
1-4	**8**	**6**	**4**
2-3	**6**	**1**	**5**
2-5	**3**	**1**	**2**
3-5	**7**	**4**	**7**
4-5	**3**	**1**	**2**

Solution: The network diagram is:



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2- Wayne L. Winston, Operations Research Applications and Algorithms, 1991.

3- I.A.Osinuga, **MTS 363 - OPERATIONS RESEARCH**.

4- Richard Bronson, Govindasami Naadimuthu, "**Operations Research**", schaum's outline series, second edition, McGraw – Hill, 1997.

5- Rogelio Acuña, **"Analysis of project planning using CPM and PERT",** Spring 2010.